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DETERMINATION OF NORTHEAST ASIA'S HIGHEST PEAK (Mt. JADE) BY DIRECT LEVELLING

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ABSTRACT

This paper demonstrates the results and problems in determining the height of Mt. Jade by direct levelling. Mt Jade is the highest peak in northeast Asia. GPS levelling and trigonometricl levelling were also made to validate the result of direct levelling. For GPS levelling a new geoid model is constructed using Taiwan's latest gravity database and elevation model. The accuracy of the geoid model ranges from cm in coastal plains to dm in high mountains. Helmert deflections of the vertical derived from this geoid model improve the accuracy of trigonometric levelling. Gravity data at benchmarks were collected to compute orthometric corrections for the heights from direct levelling. The Poincaré-Prey reduction and the modified Mader reduction of mean gravity yield orthometric corrections that differ by up to dm near Mt. Jade. The Helmert orthometric height of Mt. Jade determined in this work is 3951.798 m, with a 72-mm commission error. The problem of using a rigorous orthometric height is discussed.

KEYWORDS. GPS. Pseudolite. Obstructed sky. Positioning. Precision.

INTRODUCTION

Mt. Jade, or "Yushan" in Chinese, is the highest peak in northeast Asia. The height of Mt. Jade is a public concern in Taiwan and east Asia. Existing literature about the height shows that the published values vary from 3950 m to 4145 m, depending on the method used. Currently, the officially published height of Mt. Jade is 3952.382 m above mean sea level at Keelung, which was issued by the Ministry of the Interior (MOI) of Taiwan in 1978. This value was derived from trigonometric levelling and is in theory in the Helmert orthometric height system. According to various sources, the height systems used before 1978 are not even clear. By today's technical standards, some of the methods used to obtain the heights before 1978 are too crude to give meaningful results. Therefore, to the majority of the public in east Asia, the exact height of Mt. Jade remains a mystery. Furthermore, after the Chi-Chi Earthquake, which measured 7.3 on the Richter scale and caused a major damage to properties and lives [**16**], the height of Mt. Jade may have changed significantly and a precise determination is needed.

The topography over the Mt. Jade area is very rugged and in such terrain as the Mt. Jade and Himalaya areas, GPS levelling is commonly used for orthometric height determination. For example, [**9**] used a gravimetric geoid model of Taiwan and GPS-derived ellipsoidal heights at four first-order levelling benchmarks to compute the height of Mt. Jade. Banerjee in [**3**] used GPS levelling to determine the height of Ladak in the northwest Himalaya. [**7**] determined the orthometric height of Mt. Fuji

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using GPS levelling and direct levelling. [**17**] reviewed the methods and accuracies for determining the height of Mt. Everest. According to [**17**], the methods used to determine the height of Mt. Everest during 1843-1999 include trigonometric levelling and GPS levelling, and the measurements started from nearby benchmarks whose orthometric heights are given. A typical problem in GPS levelling in a rugged terrain is lack of a local geoid model that is compatible in accuracy and resolution with GPS-derived ellipsoidal heights. This problem is mainly due to lack of dense gravity and elevation data. Based on the current status of gravity and elevation data over the Mt. Jade area [**11**], direct levelling seems to be the only reliable method to determine the height of Mt. Jade.

Since 2000, the MOI of Taiwan has been conducting nationwide precision levelling and gravity surveying campaigns. A first-order benchmark has been deployed on a provincial route near Mt. Jade, and it is only 14.5 km to its summit along the hiking track. Because of the need of orthometric corrections to the heights from direct levelling, gravity data were also collected. For cross validation, it is also decide to collect GPS and trigonometric levelling data.

THE HEIGHT SYSTEM OF TAIWAN

In 2003, a new first-order vertical network of Taiwan was established. The benchmarks are deployed in major provincial routes of Taiwan (Figure 2). The differential heights in the network were determined by precision levelling that demands a maximum permissible misclosure of $2.5\sqrt{K}$ mm, where K is the distance in km between two neighboring benchmarks. The geodetic coordinates of the benchmarks were determined by GPS. Each benchmark was occupied for at least three hours and the coordinates were computed using the precise GPS ephemerides. In this new network, gravity values at the benchmarks were measured to compute orthometric corrections based on the Poincaré-Prey reduction of mean gravity [**8** p. 167], thus the resulting height is the Helmert orthometric height [**1**], [**5**], [**11**].

The zero point of Taiwan's current height system is mean sea level at Keelung, located in northern Taiwan. This zero point is derived from the sea level records of a tide gauge at the Keelung Harbor, which span more than 50 years. Due to a lack of knowledge of the sea surface topography, it is understood that the origin of Taiwan's height system defined in this way will not be on the geoid. Therefore, all the Helmert "orthometric heights" of the benchmarks in the first-order vertical network on the Keelung datum deviate from the "true" orthometric heights by an amount that is the sea surface topography at Keelung, as well as other systematic errors in direct Levelling and orthometric corrections.

A NEW GEOID MODEL IN TAIWAN FOR GPS LEVELLING

To aid GPS levelling in the Mt. Jade area, a local geoid model of Taiwan was constructed. We used all possible land and marine gravity data and altimeter data around Taiwan to compute a new geoid model. The altimeter data are from the Seasat, Geosat, TOPEX/POSEIDON and ERS-1 missions. Along-track geoid gradients derived from altimetry observations were used for geoid computations. Figure 2 shows the distribution of gravity and altimeter data used in the geoid modelling. The method of least-squares collocation [**13**] was used to compute the geoid model. The standard remove-restore procedure was used. EGM96 [**12**] to degree 360 was adopted as the reference field, and the short wavelength component was accounted for by the residual terrain model (RTM) [**6**]. First, residual gravity anomalies and residual geoid gradients were computed by

$$\Delta g_{res} = \Delta g - \Delta g_{EGM96} - \Delta g_{RTM} \tag{1}$$

$$e_{res} = e - e_{EGM96} - e_{RTM} \tag{2}$$

where $\Delta g, \Delta g_{EGM96}$ and Δg_{RTM} are the observed, EGM96- and RTM-contributed gravity anomalies, respectively, and e, e_{egm96} and e_{RTM} are the raw, EGM96- and RTM-contributed geoid gradients, respectively. The residual geoid was computed by

$$N_{res} = \begin{pmatrix} C_{N\Delta g} & C_{ne} \end{pmatrix} \begin{pmatrix} C_{\Delta g} + D_{\Delta g} & C_{\Delta ge} \\ C_{e\Delta g} & C_e + D_e \end{pmatrix}^{-1} \begin{pmatrix} \Delta g_{res} \\ e_{res} \end{pmatrix} \tag{3}$$

where $C_{N\Delta g}$, C_{ne}, $C_{\Delta g}$, $C_{\Delta ge}$, and C_e are the covariance matrices for geoid-gravity anomaly, geoid-geoid gradient, gravity anomaly-gravity anomaly, gravity anomaly-geoid gradient and geoid gradient-geoid gradient, respectively. These covariance matrices were computed based on the error degree variances of EGM96 from degree 2 to 360 and the modelled degree variances of [**15**] from degree 361 to infinity. Methods to construct these covariance functions and matrices are described in, e.g., [**13**]. The final geoidal height is computed by $N = N_{res} + N_{EGM96} + N_{RTM}$, where N_{EGM96} and N_{RTM} are the EGM96- and RTM-contributed geoidal heights, respectively.

Ellipsoidal heights derived from 24-hour GPS observations on four levelling routes (see Figure 1 for the locations) of the first-order network were used to evaluate the accuracy of the geoid model. The terrain near these four levelling routes is quite different, ranging from coastal plains to high mountains. The formal standard errors of ellipsoidal heights and the orthometric heights, which are available in the output of the corresponding adjustment computations, are all below 1 cm. An "observed" geoidal height at a benchmark can be obtained by subtracting the orthometric height from the ellipsoidal height. However, as mentioned before, the heights of the benchmarks at the four levelling routes are in the Helmert orthometric height system and tied to mean sea level at Keelung. Therefore, a geoidal height in this case will also contain unknown error arising from the uncertainty in orthometric correction and sea surface topography (SST). Table 1 shows a comparison of the observed and modeled geoidal heights. Because differenced geoidal heights are less sensitive to long wavelength error of the modeled geoid and SST bias, we used the standard deviations in Table 1 as the criterion of geoid quality. As seen in Table 1, the standard deviation of differences increases with elevation. At the levelling route across the Central Range, the highest elevation reaches 3500 m and here the difference is the largest.

Table 1: *Statistics of differences between "observed" and modeled geoidal heights along four first-order levelling routes (units: m)*

Levelling route	Mean elevation	Mean	Std. dev.	Maximum	Minimum	RMS
1 (north Taiwan)	13	0.422	0.024	0.450	0.367	0.423
2 (east Taiwan)	123	0.259	0.130	0.424	0.068	0.287
Central cross-island highway	1519	0.012	0.350	0.464	-0.466	0.332
Southern cross-island highway	402	0.167	0.181	0.280	-0.298	0.239

The elevations near Mt. Jade are all above 2700 m, so it would be expected that the uncertainty of the geoid model will reach the dm level. The inferior accuracy of the geoid model over mountainous areas is mainly caused by the poor coverage of gravity data and the high frequency gravity variation.

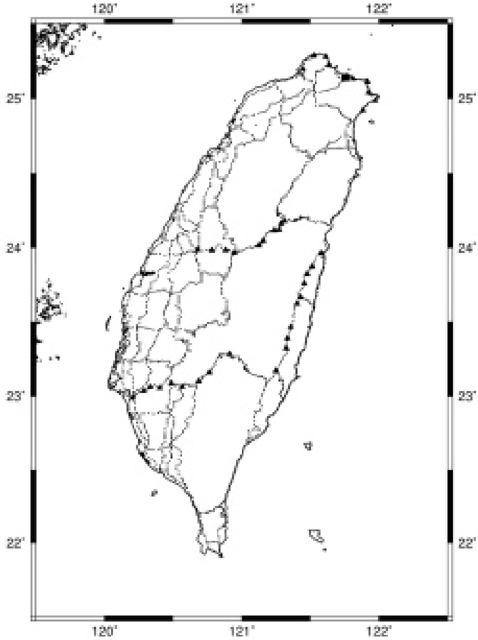

Fig. 1: Distribution of benchmarks in the first-order vertical network of Taiwan (completed in 2003). The triangles represent benchmarks where modeled and "observed" geoidal heights are compared. Square represents the origin of network at Keelung

DATA COLLECTION AND REDUCTION

The surveying field work

Three surveying methods were used to determine and validate the height of Mt. Jade: direct levelling, GPS levelling and trigonometric levelling. The levelling route, which is one of the hiking tracks to Mt. Jade, begins with a first-order levelling benchmark, X121, and ends with the summit of Mt. Jade, which is numbered S026 in Taiwan's first order satellite control network.

From X121 to S026, a total of 17 temporary benchmarks, named YS01, …, YS17, were deployed at a mean along-track spacing of 500 m. Most of the temporary benchmarks were selected at sites with an open view in order to facilitate GPS determinations of coordinates and trigonometric levelling. The levelling route to Mt. Jade is narrow, rugged and in most cases very unfriendly. The slopes in many sections of the route are large enough to make the setup of the level nearly impossible. For example, the section from YS17 to S026 (Mt. Jade) is a steep ascent of more than 320 m in a 1.2-km along-track distance. From a point about 300 m to S026, the surface is filled with loose gravel and it is even difficult to walk over the surface without the assistance of iron chains that were placed to help Mt. Jade climbers.

Direct levelling

Direct levelling is considered the most precise tool for the Mt. Jade heighting. A Leica NA2002 digital level was used to determine the height differences from X121 to S026. Because of the rough terrain, we adopted the criterion that the misclosure of the forward and backward height differences between two neighboring benchmarks should be less than 20 mm $\sqrt{K}$ on paved route (only a small section of the levelling route is

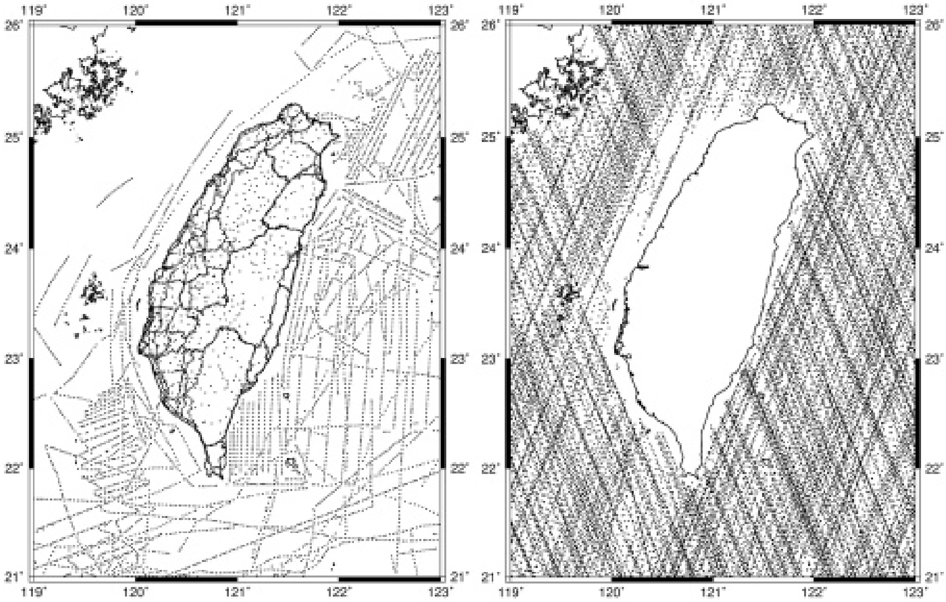

Fig. 2: Distribution of terrestrial gravity data (left) and altimeter data for the geoid modelling

paved) and 30 mm $\sqrt{K}$ on unpaved route. Because of logistic problems and bad weather conditions, direct levelling was carried out in four different campaigns, starting from June 17, 2003 to November 26, 2003. The result shows that all the misclosures are below the 20 mm or 30 mm $\sqrt{K}$ error bound. One exception occurs between YS17 and S026 (the total distance: 1.2 km), where the misclosure is 57 mm. A maximum misclosure of 16 mm was found in one of the sub-sections between these two points.

Gravity survey

For orthometric corrections, relative gravity measurements were collected at all the surveying monuments (X121, YS01 to YS17, and S026) using a LaCoste and Romberg type G gravimeter (serial number G838). The gravity value at X121 has been determined when establishing the first-order vertical network (described earlier). The nominal instrument noise of this gravimeter is 0.04 mgal, which was confirmed by Hwang et al. [**10**]. The gravity survey procedure was similar to that of direct levelling. In this procedure, relative gravity values on consecutive benchmarks were collected in a closed loop. Such a loop forms a session of gravity survey and the measuring time cannot exceed two hours in order to reduce the effect gravimeter drift and other systematic errors. All the raw gravity measurements were then corrected for solid Earth tide, air pressure and ocean loading effects. The misclosure of relative gravity values for X121-S026 is below 1 mgal and is considered acceptable under this special condition. Finally, the absolute gravity values at all surveying monuments were computed with the gravity at X121 as the starting value. Figure 3 shows gravity values relative to heights along the levelling route. Gravity value decreases almost linearly with height: the correlation coefficient between height and gravity is -0.998 and the gradient of gravity is -0.214 mgal/m. This linear relationship is typical and enables the use of a simplified formula for orthometric correction; see [**11**].

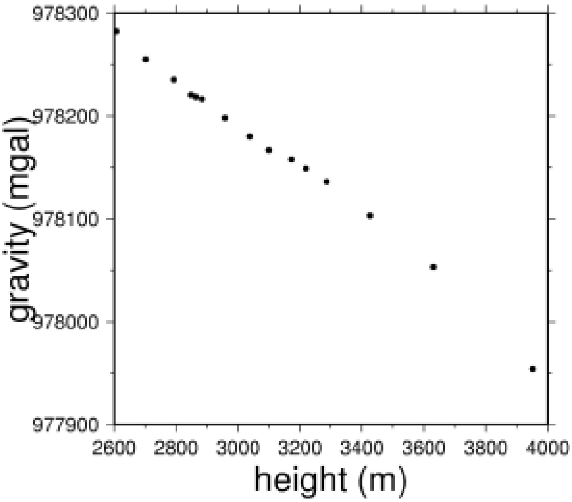

Fig. 3: Gravity values vs. elevations at benchmarks along the levelling route

GPS survey

The purpose of GPS survey is twofold. The first one is to determine the height of Mt. Jade by GPS levelling and the second is to cross-validate the result of direct levelling. For the first purpose, GPS data were collected at six first-order benchmarks (L068, L069, L071, L074, L075 and X121) and S026 (Mt. Jade) in a continuous 24-hour session. These six benchmarks are located on the Provincial Route No. 21 and are about 7-9 km in horizontal distance to Mt. Jade. The GPS measurements are used to determine the precise ellipsoidal height differences between the six benchmarks and S026, and then to compute the orthometric height of S026 with the aid of a local geoid model (see below under Results).

For the second purpose, GPS survey was conducted at benchmarks where direct levelling is difficult to carry out. In all cases, Trimble GPS receivers were used and the coordinates of the benchmarks and Mt. Jade were determined using the Bernese Version 4.2 GPS software and the precise GPS ephemerides from the IGS.

Trigonometric levelling

Trigonometric levelling was done at selected levelling sections where the terrain is complicated, including sections YS04-YS05, YS11-YS12, YS16-YS17, and YS17-S026. This is for a cross validation of the results from direct levelling and GPS. To reduce the effects of atmospheric refraction, reciprocal observations were made in all cases. In theory, trigonometric levelling yields ellipsoidal height differences, equivalent to those obtained by GPS. Because the distance between any two participating points are relatively short (less than 1 km), the ellipsoidal height difference, Δh, can be computed by the formula

$$\Delta h = s \cos Z + i - z + C_r + C_c \tag{4}$$

where s, Z, i, z, C_r, and C_c are the slant distance, zenith distance, height of the theodolite, height of the target and the corrections for atmospheric refraction and Earth curvature, respectively. We used the approximate formulae $C_r = 0.16(\mathrm{s}^2/2R)$ and $C_c = \mathrm{s}^2/2R$ [**2**], where $R \approx 6371\,\mathrm{km}$ is the mean radius of the Earth. The EDM-measured slant distance was corrected for the atmospheric refraction, using air pressure, temperature and water vapor contents [**2** Sect. 4.37]. In the Mt. Jade area the atmospheric corrections for the slant distances range from 70 parts per million (ppm) to 90 ppm, which translate into a maximum correction of about 6 cm. These large atmospheric corrections were caused by the low pressures (lowest: 650 hPa) and the low temperatures (lowest: $12\,°\mathrm{C}$) at the measuring times, compared to the EDM's normal working condition at air pressure=1013 hPa and temperature = $25\,°\mathrm{C}$.

According to Zhang [**17**], neglecting deflections of the vertical has introduced errors in determining the height of Mt. Everest by trigonometric levelling. The terrain in the Mt. Jade area is very rugged and similar to that of Mt. Everest, thus Helmert deflections of the vertical need to be considered. The observed zenith angles at two end points P_1 and P_2, which are based on local plumblines, need to be reduced to zenith angles based on the corresponding ellipsoidal normals. Let Z_1' and Z_2' be the observed zenith angles. The correction for the effect of deflection of the vertical is [**8**, p.174]

$$
\begin{aligned}
Z_1 &= Z_1' + \xi_1 \cos\alpha + \eta_1 \sin\alpha \\
Z_2 &= Z_2' - \xi_2 \cos\alpha - \eta_2 \sin\alpha
\end{aligned}
\tag{5}
$$

where α is the azimuth from P_1 to P_2, ξ_1 and η_1 and ξ_2 and η_2 are the north-south and west-east components of the deflections of the vertical at P_1 and P_2, respectively. Helmert deflections of the vertical can be obtained by numerically differentiating the geoidal height (see Section 3) using the relationships

$$
\xi = -\frac{\partial N}{R\partial\phi}, \eta = -\frac{\partial N}{R\cos\phi\partial\lambda} \tag{6}
$$

where λ and ϕ are longitude and latitude.

ORTHOMETRIC CORRECTION. POINCARE-PREY REDUCTION vs MODIFIED MADER REDUCTION

We used two different methods to compute orthometric corrections, which will be applied to the height differences from direct levelling. [**11**] shows that, if gravity varies linearly with height between two neighboring benchmarks, A and B, the orthometric correction can be computed by

$$
OC_{AB} = \frac{1}{\bar{g}_B}\left(\frac{g_A + g_B}{2} - \bar{g}_B\right)\Delta n_{AB} + H_A\left(\frac{\bar{g}_A}{\bar{g}_B} - 1\right) \tag{7}
$$

where Δn_{AB} is the leveled height difference between A and B, H_A is the height at A, g_A and g_B are surface gravity values, and $\bar{g}_A$ and $\bar{g}_B$ are the mean gravity values along the plumblines at A and B. The difference in the two methods lies in the methods for computing the mean gravity value along the plumbline. In the first method, the mean gravity is obtained by the Poincaré-Prey reduction [**8** p. 167]

$$\bar{g} = g + 0.0424H \tag{8}$$

where g is the surface gravity value and H is the approximate orthometric height. The orthometric correction computed in this way is called OC1 in this paper, and the resulting height is called the Helmert orthometric height. In the second method, the mean gravity is computed using the modified Mader reduction. The difference between the Mader reduction and the modified Mader reduction is that the latter takes into account the terrain effect, actual vertical gravity gradients and density variations [**11**]. In this paper, the density mode used in this paper is from [**4**], which is on a $5' \times 5'$ grid. More detail of the modified Mader reduction can be found in [**11**]. A maximum vertical gravity gradient of 0.3777 mgal/m is found near Mt. Jade, and this is significantly different from the normal gravity gradient of 0.3086 mgal/m. With the height of Mt. Jade equal to about 3952 m, this discrepancy in gravity gradient will introduce an error of 3952×0.0691 =273 mgal in the free-air reduction of gravity. An alternative and rigorous treatment of mean gravity along the plumbline can be found in Tenzer (2004).The orthometric correction based on the modified Mader reduction is called OC2 in this paper.

The terrain in the Mt. Jade area features deep valleys and sharply rising peaks. The assumption of a uniform Bouguer plate under the surface of a computing point in the Poincaré-Prey reduction can hardly hold and will certainly lead to a significant error in computing the mean gravity. On the other hand, the modified Mader reduction considers the actual terrain, as well as gravity gradient and density variation, so it should yield a more realistic mean gravity than the Poincaré-Prey reduction does. Figure 4 shows a comparison of the two methods of orthometric reduction along the levelling route. The difference of the two orthometric corrections near the summit of Mt. Jade is at the dm level. Due to terrain, in the last three levelling sections, namely, YS15-16, YS16-17, and YS17-S026, the trends of OC1 and OC2 are opposite: OC1 increases with height, while OC2 decreases with height. A further comparison of OC1 and OC2 will be given below under Results.

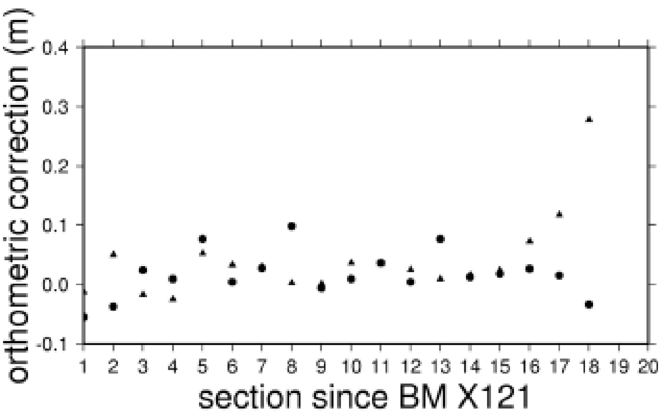

Fig. 4: Orthometric corrections based on the Prey reduction (triangle) and the modified Mader reduction (circle)

RESULT AND ANALYSIS

Comparison of direct levelling, GPS levelling and trigonometric levelling

At selected sections along the levelling route, GPS levelling and trigonometric levelling also exist and their results can be used to validate the result of direct

levelling. Table 2 shows a comparison of height differences from the three methods at common sections. From the comparison in Table 2, we find that:
1) The differences between OC1 and OC2 at YS16-YS17 and YS17-S026 are 0.102 m and 0.312 m, respectively. These differences are very large compared to the error of direct levelling. The differences between OC1 and OC2 at other sub-sections are relatively small.
2) At X121-YS04, the ellipsoidal height difference from GPS deviates from the OC2-corrected height difference (from direct levelling) by 0.337 m, but the difference is reduced to 0.089 m if ellipsoidal height difference is corrected for geoidal height difference. This shows that the geoid model performs well at this section.
3) At YS16-YS17 and YS17-S026, the OC1-corrected height differences deviate significantly from those from GPS and trigonometric levelling. These large differences should be caused by OC1, which is only an approximate correction. On the other hand, the OC2-corrected height differences agree well with the GPS results. Therefore, OC2 performs better than OC1 in the Mt. Jade area.
4) The ellipsoidal heights from deflection of the vertical-corrected trigonometric levelling agree better with those derived from GPS as compared to those without deflection of the vertical correction.

A relatively large discrepancy in ellipsoidal height from GPS and trigonometric levelling is found at the last section (YS17-S026), and this is due to large vertical angles and bad weather conditions. Table 2 also lists the averaged deflections of the vertical of the end points at the common sections. Because the Central Range lies mainly in the north-south direction, the west-east components of the deflection of the vertical are generally larger than the north-south components. Furthermore, adding deflection of the vertical in trigonometric levelling also reduces the discrepancies between the forward and backward heights, which are at the cm level.

Table 2: *Comparison of height differences from direct levelling, GPS levelling and trigonometric levelling*

Section	DL (m)	DL+OC1 (m)	DL+OC2 (m)	Δh (m)	ΔN (m)	TL1 (m)	TL2 (m)	Aver.DOV for north/ East (arc- sec)
X121-YS04	-3.274	-3.279	-3.333	-2.996	-3.188			
YS04-YS05	94.241	94.293	94.316	94.301	94.224	94.223	94.301	15/27
YS05-YS06	91.501	91.534	91.505	91.562	91.486			
YS10-YS11	79.031	79.065	79.067	79.117	79.015			
YS11-YS12	62.627	62.652	62.631	62.635	62.623	62.624	62.635	9/38
YS16-YS17	205.068	205.185	205.083	205.080	205.025	205.033	205.088	16/30
YS17-S026	320.403	320.681	320.369	320.311	320.290	320.339	320.359	15/28

DL: height difference from direct levelling
Δh : ellipsoidal height difference from GPS
ΔN : ellipsoidal height difference minus geoidal height difference
TL1 and TL2: height differences from trigonometric levelling without and with corrections for the effect of deflection of the vertical
DOV: deflection of the vertical
OC1, OC2: orthometric corrections using Poincaré-Prey reduction and modified Mader reduction

The orthometric height of Mt. Jade from direct levelling

The orthometric height of Mt. Jade is obtained by summing the orthometric height of X121 (in the Helmert orthometric height system) and the orthometric height difference (height differences corrected for OC1) between X121 and Mt. Jade. This leads to the conclusion that the Helmert orthometric height of Mt. Jade is 3951.798 m (at the top of the survey monument). If the more precise orthometric correction OC2 is used, then the height of Mt. Jade is 3951.376 m. In theory, one could adopt this value as the orthometric height of Mt. Jade. There is an immediate drawback in using this value: it is not uniquely defined as the Helmert orthometric height because it depends on the DEM model, the density model and the reduction procedure used.

The new Helmert orthometric height of Mt. Jade is 0.584 m shorter than the latest published height of 3952.382 m, which was derived from trigonometric levelling. This does not mean that Mt. Jade has experienced a vertical site movement of 0.584 m from 1978 to 2003. This difference is mainly caused by different methods used. The measurements used to obtain this new height are more precise and have been validated by other methods in comparison to previous ones. Compared to previous results, the result described in this work is characterized by the fact that (1) Direct levelling is used from X121 to Mt. Jade, (2) orthometric correction based on measured gravity values has been applied, and (3) the definition of the new height of Mt. Jade is clear and conforms to the Taiwanese height system (Helmert height system) of the newly established first-order vertical network of Taiwan

Because the levelling route from X121 to Mt. Jade is an open route rather than a closed network, it is not possible to perform a rigorous adjustment of the OC-corrected height differences and to estimate rigorously the standard errors of the heights of the 17 temporary BMs and Mt. Jade. However, we do have a rigorous check of the misclosure of the forward and backward height differences at each levelling section. As described before, the misclosures of forward and backward height differences range from 0 to 14 mm, except at YS17-S026, where the misclosure is 57 mm due to the extremely difficult levelling condition.

Using the levelling misclosures, we adopt the following three estimates of the standard error of the height difference between X121 and Mt. Jade:

1) The standard error is estimated as the squared root of the sum of the squared misclosures of all sections. The result is 64 mm.

2) Same as (1), but replace misclosures less than 5 mm by 5 mm. The result is 65 mm.

3) Same as (1), but replace misclosures less than 10 mm by 10 mm. The result is 72 mm.

Therefore, it is estimated that the standard error of direct levelling between X121 to Mt. Jade range from 65 mm to 72 mm. According to error propagation, the error of Mt. Jade height is due to the error of X121 height (7 mm), the error of the leveled height difference (72 mm), and the error of OC. Assuming that $\sigma_{g_A} = \sigma_{g_B}$=0.2 mgal [10], by Eq. (7) the standard error of OC (by the Poincaré-Prey reduction) is 3.73 mm, which is relatively small compared to the error of direct levelling. It is understood that the OC error of 3.73 mm is only the commission error in using the Poincaré-Prey reduction to compute the OC. The model error of the Poincaré-Prey reduction is unknown, but could be large due to the assumption used in this reduction (see the discussion above). Again, the standard error of 7 mm for X121's Helmert orthometric height is only the commission error, and the model error due to the Poincaré-Prey reduction at X121 is not known.

In summary, by setting the standard errors of the X121 height, the height difference from direct levelling and the OC to be 7 mm, 72 mm and 4 mm, respectively, the estimated standard error of the new height of Mt. Jade is $\sqrt{7^2+72^2+4^2}=72\,\mathrm{mm}$. We decide to adopt 72 mm as the standard error for the new Helmert orthometric height of Mt. Jade (3951.798 m) obtained in this work.

The orthometric height of Mt Jade from GPS Levelling

Despite the considerable effort to obtain the Helmert orthometric height of Mt. Jade, there is still one question left to be answered: how much does this new Helmert orthometric height of Mt. Jade (3951.798 m) deviate from the true orthometric height (the true distance along the plumbline from the summit of Mt. Jade to the geoid)? To investigate the "true" orthometric height of Mt. Jade, we used the geoid model and the 24-hour GPS observations on six benchmarks (see above) to compute the orthometric height differences between Mt. Jade and the six benchmarks (Table 3). The orthometric heights of Mt. Jade from these six benchmarks differ from each other. The geoidal difference increases with increasing elevation difference. Table 3 shows that, the larger the height difference between Mt. Jade and a benchmark, the larger the difference between the GPS-derived and the direct levelling-derived orthometric heights.

The most adequate orthometric height in Table 3 for comparison with direct levelling-derived orthometric height is the one from X121, i.e., 3950.599 m. In this case, the differences in orthometric height from direct levelling and GPS levelling results are 1.199 m (Helmert height) and 0.777 m (height with the modified Mader reduction). If we adopt the mean of the six orthometric heights as the GPS-derived orthometric height of Mt. Jade, the corresponding differences are 1.361 m and 0.939 m. In these two cases, the GPS-derived orthometric heights of Mt. Jade has a better agreement with the OC2-corrected direct levelling result than the OC1-corrected one, but the differences (0.777 m and 0.939 m) are still large compared to the error of direct levelling.

The comparison between the modeled gravimetric and the "measured" geoidal heights in Table 1 indicates that, in a terrain similar to that over the Mt. Jade area (see the Central cross-island highway in Table 1), the accuracy of the gravimetric geoid is about 35 cm. Also, Table 2 has shown that, along the levelling route to Mt. Jade, the maximum discrepancy between the gravimetric and the "measured" geoidal heights is only 10 cm (see the columns DL+OC2 and ΔN in Table 3). Therefore, the differences of 1.199 m and 0.777 m between the orthometric heights of Mt. Jade derived from direct levelling and from GPS levelling are caused in part by the errors in Poincaré-Prey reduction and the modified Mader reduction, respectively. From this analysis, it is seen that comparing the difference between orthometric heights from direct levelling and from GPS levelling in high mountains can lead to an ambiguous conclusion: it is not clear that the difference is due to improper orthometric correction or due to the error in the geoid model, or both.

CONCLUSIONS

Based on the result of direct levelling and the given height at benchmark X121, Mt. Jade's Helmert orthometric height is 3951.798 m, with a 72-mm commission error. This commission error does not fully represent the deviation of Mt Jade's new height (3951.798 m) from its "true" orthometric height, which is the distance from the summit of Mt. Jade to the geoid along the plumbline. In other words, if the modelling errors of the X121 height (Helmert orthometric height) and the orthometric correction (by the Poincaré-Prey reduction) are taken into account, the total error of this newly determined height will be more than 72 mm.

This paper also shows that comparing the "true" geoidal heights from direct levelling and the modeled geoidal heights in high mountains may lead to false conclusions. This is because the difference may come from the orthometric correction (for direct levelling) and/or the geoid model error. On the other hand, if a highly accurate geoid model is available for GPS levelling in a high mountain area like Mt. Jade, it can be used to evaluate the accuracy of an orthometric correction. Given the current spatial resolution of gravity and DEM data sets in Taiwan, a highly accurate geoid model cannot be obtained now. However, it is expected that an island wide airborne gravity survey will be conducted in 2004 and 2005, and a high-resolution (on a 5-m grid), high-accuracy (at metre level) elevation model will be completed in 2005. These two data sets are expected to improve the accuracies of the medium and short wavelength components of Taiwan's local geoid. The improvement in the long wavelength component of the geoid is expected to come from the gravity models associated with the CHAMP, GRACE and GOCE satellite missions.

Table 3: *The orthometric heights of Mt. Jade derived from six first-order benchmarks (unit: m)*

Benchmark	Ellipsoidal height	OH	Δh	ΔN	ΔH	OH of Mt. Jade at BM
L068	1845.786	1819.109	2132.668	1.503	2131.165	3950.274
L069	1946.364	1919.902	2032.090	1.746	2030.344	3950.246
L071	2185.114	2158.776	1793.340	1.761	1791.579	3950.355
L074	2482.210	2455.838	1496.244	1.520	1494.724	3950.562
L075	2556.648	2530.200	1421.806	1.415	1420.391	3950.591
X121	2636.222	2609.609	1342.232	1.242	1340.990	3950.599

Δh : ellipsoidal height difference (Mt. Jade –benchmark)
ΔN : geoidal height difference (Mt. Jade –benchmark)
ΔH : orthometric height difference, $\Delta H = \Delta h - \Delta N$
OH: orthometric height, BM; Benchmark

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