

# A detailed gravity field over the Reykjanes Ridge from Seasat, Geosat, ERS-1 and TOPEX/POSEIDON altimetry and shipborne gravity

Cheinway Hwang<sup>1</sup>, Barry Parsons, Theresa Strange and Andrew Bingham

Department of Earth Sciences, Oxford University, Oxford, UK

**Abstract.** The method of least-squares collocation (LSC) was used to construct a detailed gravity field over the Reykjanes Ridge using Seasat, Geosat, ERS-1, TOPEX/POSEIDON altimeter data and shipborne gravity data. Mean altimeter-derived geoid gradients, along with their standard deviations, were used in order to avoid cross-over adjustment of sea surface heights. The ship data were adjusted to a satellite-only gravity field using a quadratic polynomial in time and then merged with the satellite data to produce a combined gravity field. The necessary covariance functions were derived using the law of covariance propagation. In a test case, a comparison between a combined gravity field and ship data not included in the calculation yielded an rms difference of 2.31 mgals. The combined gravity field contains a wealth of short-wavelength information and clearly reveals the detailed tectonic structures associated with the Reykjanes Ridge.

## Introduction

Over the past two decades, Geos-3, Seasat and Geosat altimeter data have provided valuable information for geophysical, geodetic, and oceanographic studies. The current ERS-1 and TOPEX/POSEIDON (T/P) satellite missions have increased substantially the coverage of high-precision estimates of the mean sea-surface. However, apart from the Geosat Geodetic Mission, which has a uniform resolution of 12 to 18 km (McAdoo and Marks, 1992), the inter-track spacing of the other altimeters is still considerably larger than the along-track sampling interval, and the shorter-wavelength features are far from uniformly sampled. In many regions of tectonic interest, shipborne gravity data is available containing abundant, additional short-wavelength information. The technique of least-squares collocation (LSC) allows both altimeter and ship gravity data to be used together to produce a gravity field with as much short-wavelength detail as possible. As an example of this method, we have constructed a gravity field over the Reykjanes Ridge in the north Atlantic Ocean

from Seasat, Geosat, ERS-1, T/P altimeter data and all the ship gravity data for the area. In previous studies using least squares collocation, e.g., Rapp and Basic (1992), altimeter-derived geoid heights were used. In the present study, we use geoid gradients, thus avoiding cross-over adjustments of sea surface heights (ssh) needed for removing the bias part of orbital errors (e.g., Knudsen, 1987). Sandwell (1992, p.438) also has some discussions on the advantage of using along-track geoid gradients over ssh. Gravity anomalies have been derived from Geosat geoid gradients by McAdoo and Marks (1992) and Sandwell (1992) using a Fourier transform method. One advantage of least squares collocation is that geoid gradients along satellite tracks with different inclinations can be included without any change in the method. This paper also presents a procedure for correcting ship gravity data for long-wavelength errors by first comparing it with a gravity field derived using altimeter data alone.

## The Altimeter Data

The altimeter data to be used are along-track geoid gradients derived from the Seasat, Geosat, ERS-1 and T/P sea surface heights. The typical ground track coverage of the altimeter data is illustrated in Fig. 1. The Seasat gradients were obtained by numerical differentiation of sea-surface heights in the globally adjusted data set described by Rapp (1982). The Geosat Exact Repeat Mission (ERM), ERS-1 and T/P have ground tracks that are repeated every 17, 35 and 10 days respectively. The geoid gradients in these cases were obtained by differencing along-track, and then averaging over a number of passes along a given ascending or descending ground track. Geosat ERM gradients were averaged over 60 repeat cycles (Cheney et al. 1987). For ERS-1 the gradients were averaged over the first 5 cycles, using the Interim Geophysical Data Records (IGDR) produced by Cheney et al. (1991). The 10-day T/P gradients are from AVISO (1992) and averaged over the first 10 cycles. As well as average geoid gradients, estimates of standard errors in the mean are obtained and used in the LSC inversion, which provides an optimal way of combining data with varying quality. The averaging process is able to improve the signal-to-noise ratio of the altimeter data; in the case of the average Geosat data an improvement factor of up to 7.5 was obtained (Strange, 1991). The ERS-1 and T/P mean gradients have larger standard deviations due to the

<sup>1</sup>Now at Department of Civil Engineering, National Chiao Tung University, Hsinchu, Taiwan

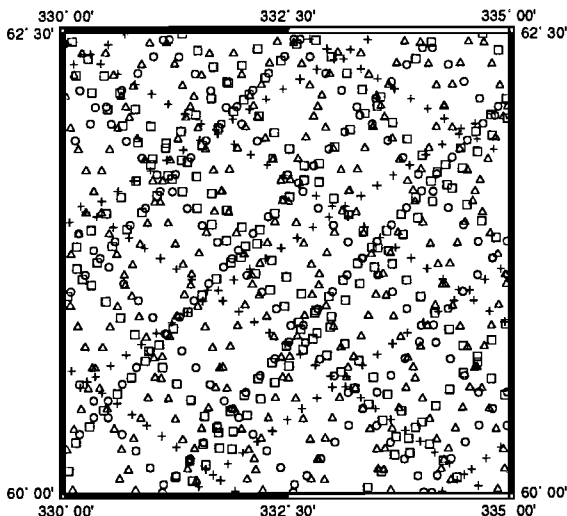
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smaller numbers of tracks used and shorter intervals of time, which may not be sufficient to smooth completely the time varying component of the sea surface. Table 1 summarizes the percentages of the standard deviations within  $1 \mu\text{rad}$  through  $5 \mu\text{rads}$  over the Reykjanes Ridge for Geosat, ERS-1 and T/P. For Seasat geoid gradients, an uniform standard deviation of  $10 \mu\text{rads}$  was assigned to the gradient data using the approximate formula  $\sigma_g^2 = (\sigma_1^2 + \sigma_2^2)/d^2 = 2\sigma^2/d^2$ , where  $\sigma = 5 \text{ cm}$  is Seasat's noise level, and  $d = 6.73 \text{ km}$  is the approximate along-track spacing. The rms values of the geoid gradients of Seasat, Geosat, ERS-1, and T/P are 20.70, 17.64, 14.86, 19.65  $\mu\text{rads}$  respectively and are greater than their noises.

### The Shipborne Gravity Data

The ship gravity data are the free-air gravity anomalies in a global database compiled from the holdings of the major data centres. Fig. 2 shows the data distribution over the Reykjanes Ridge. As shown in Table 1 of Wessel and Watts (1988), shipborne gravity data are known to contain several types of errors. Their global analysis showed that the cross-over errors of the raw data have a standard deviation of 22.43 mgals, which was reduced to 13.96 mgals after an adjustment of each cruise by a bias and a linear trend with time in order to minimise crossing errors. For the LSC method it is necessary to know the accuracies of the shipborne gravity data. Assuming that Wessel and Watts' adjustment has successfully removed the systematic part of the gravity errors, the value 13.96 mgals then reflects the random part of the errors. By further assuming that the number of points with a given weight are the same, we obtain a relationship between weights of cruises given in Table A1 of Wessel and Watts (1988) and the associated standard deviations of data, as shown in Table 2. For cruises not listed in Table A1 of Wessel and Watts, the weights are assigned according to the years of data collection as follows: before 1965, the weights are identically 1; from 1965 on, the weights are increased by 1 every two years;



**Figure 1.** Typical satellite ground tracks in a  $2.5^\circ \times 5^\circ$  cell (alternate data points selected). Seasat: square, Geosat/ERM: circle, ERS-1: triangle, T/P: cross.

**Table 1.** Percentages of standard deviations for the geoid gradients within 1, 2, 3, 4 and 5  $\mu\text{rads}$  over the Reykjanes Ridge

satellite	< 1	< 2	< 3	< 4	< 5	mean
Geosat	42.64	97.66	99.27	99.55	99.74	1.12
ERS-1	8.25	23.24	42.09	59.53	72.98	3.98
T/P	0.15	1.89	16.05	44.31	71.41	4.37

after 1981, the weights are identically 10. Such an estimate of weights is based upon the ship's navigation systems which have improved over time and are the major factor governing the accuracies of the shipborne data. There are 68 ship cruises over the Reykjanes Ridge, 46 of which have over 100 gravity measurements in this area.

### Gravity Anomalies From Altimeter Data and Shipborne Gravity Data by LSC

The least squares collocation method is well documented, for example, in Moritz (1980). Following Rapp and Basic (1992) we employed a remove/restore procedure, for which the OSU91A geopotential model (Rapp et al., 1991) was used. The expressions used are:

$$\Delta g = \Delta g_{ref} + [ C_{\Delta g \epsilon}^s \quad C_{\Delta g \Delta g}^s ] \cdot$$

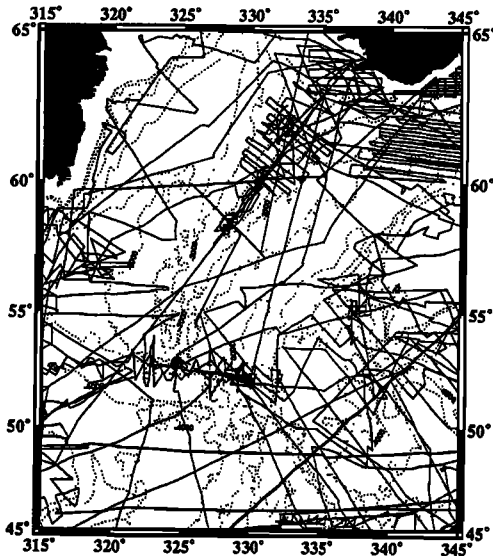
$$\begin{bmatrix} C_{\epsilon \epsilon}^o + D_\epsilon & C_{\epsilon \Delta g}^o \\ C_{\Delta g \epsilon}^o & C_{\Delta g \Delta g}^o + D_{\Delta g} \end{bmatrix}^{-1} \begin{bmatrix} \underline{\epsilon} \\ \underline{\Delta g} \end{bmatrix} \quad (1)$$

$$\sigma_{\Delta g}^2 = C_0 - [ C_{\Delta g \epsilon}^s \quad C_{\Delta g \Delta g}^s ] \cdot$$

$$\begin{bmatrix} C_{\epsilon \epsilon}^o + D_\epsilon & C_{\epsilon \Delta g}^o \\ C_{\Delta g \epsilon}^o & C_{\Delta g \Delta g}^o + D_{\Delta g} \end{bmatrix}^{-1} \begin{bmatrix} (C_{\Delta g \epsilon}^s)^T \\ (C_{\Delta g \Delta g}^s)^T \end{bmatrix} \quad (2)$$

In the above two equations,  $\underline{\epsilon}$  and  $\underline{\Delta g}$  are vectors containing residual geoid gradients and residual gravity anomalies. The covariance matrices  $C_{\epsilon \epsilon}$ ,  $C_{\Delta g \epsilon}$ , and  $C_{\Delta g \Delta g}$  are for gradient-gradient, gravity anomaly-gradient and gravity anomaly-gravity anomaly respectively, and contain the error part (from degree 2 to 360) implied by the coefficient errors of the geopotential model and the signal part (from degree 361 to infinity) implied by Tscherning/Rapp's anomaly degree variance model (Tscherning and Rapp, 1974, eqn(25A)). The superscript "s" denotes covariance matrix between the predicted value and the observables (in  $\underline{\epsilon}$ ,  $\underline{\Delta g}$ ) and "o" denotes covariance between the observables. The diagonal matrices  $D_\epsilon$  and  $D_{\Delta g}$  contain the variances of geoid gradients and shipborne gravity anomalies, respectively;  $\sigma_{\Delta g}^2$  are the *a posteriori* estimates of variances in the predicted gravity field.  $\Delta g_{ref}$  is the reference gravity anomaly computed from OSU91A model to harmonic degree 360. If ship gravity data is lacking in a prediction cell, or if a gravity field derived only from altimeter data is required, only the matrices  $C_{\Delta g \epsilon}^s$ ,  $C_{\epsilon \epsilon}^o$  and  $D_\epsilon$ , and the vector  $\underline{\epsilon}$  will be present in equation (1).

The required covariance functions can be derived using the law of covariance propagation. In particular,



**Figure 2.** Ship tracks (solid lines) and bathymetric contours (dotted lines) over the Reykjanes Ridge (contour interval: 500 meters). The thick line corresponds to the ship track for cruise c2115.

the covariance functions involving the geoid gradients and gravity anomalies between two arbitrary points P and Q are:

$$C_{\epsilon_P \epsilon_Q} = -C_{l_P l_Q} \cos(\alpha_{\epsilon_P} - \alpha_{PQ}) \cos(\alpha_{\epsilon_Q} - \alpha_{QP}) - C_{m_P m_Q} \sin(\alpha_{\epsilon_P} - \alpha_{PQ}) \sin(\alpha_{\epsilon_Q} - \alpha_{QP}) \quad (3)$$

$$C_{\Delta g_P \Delta g_Q} = \cos(\alpha_{\epsilon_Q} - \alpha_{QP}) C_{\Delta g_P l_Q} \quad (4)$$

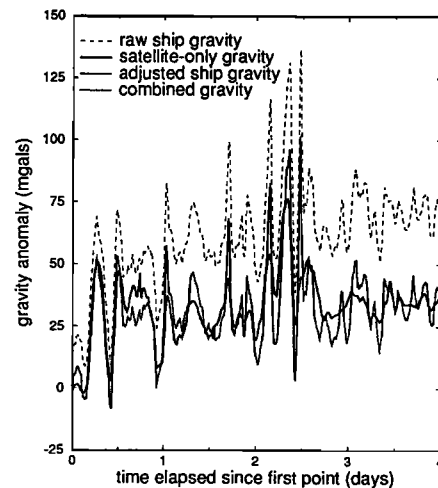
where  $\alpha_{\epsilon_P}$  and  $\alpha_{\epsilon_Q}$  are the azimuths of geoid gradients at points P and Q,  $\alpha_{PQ}$  is the azimuth from P to Q,  $C_{l_P l_Q}$  and  $C_{m_P m_Q}$  are the covariance functions for the longitudinal components and transverse components of geoid gradients respectively, and  $C_{\Delta g_P l_Q}$  is the covariance function between gravity anomaly and the transverse component of geoid gradient. The covariance functions on the right sides of (3) and (4) are dependent only on the spherical separation between P and Q and can be computed before the actual computation takes place. The derivation of (3) and (4) follows closely that given in Section 8 in Moritz (1972). The covariance functions in (3) and (4) are global and were scaled to local covariance functions (to be used in (1) and (2)) using the ratio between the variance of geoid gradients estimated in a prediction cell and the value  $C_{ll}(0) = C_{mm}(0)$ . The actual predictions were made on a  $2' \times 2'$  grid in a half degree cell. The data were

**Table 2.** Weights of ship cruises and associated standard deviations (in mgals) of shipborne gravity measurements

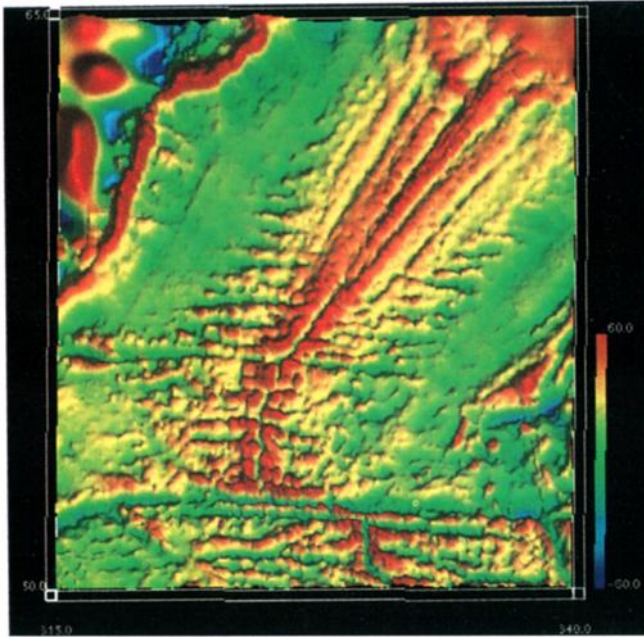
weight	std. dev.	weight	std. dev.
1	13.31	6	5.43
2	9.41	7	5.03
3	7.68	8	4.71
4	6.65	9	4.44
5	5.95	10	4.21

selected from the half degree cell and a quarter degree border about the cell.

Fig. 3 compares the gravity field derived using altimeter data alone with ship gravity along cruise c2115 (see Fig. 2), showing a long-wavelength offset between the two. The minimum dc offset found is 1.33 mgals (cruise d093b), while the maximum is 29.79 mgals (cruise c2115). Because this difference varies from cruise to cruise, it is probably due to long-wavelength errors in the ship gravity of the kind pointed out by Wessel and Watts (1988). Recent cruises show little offset. For example, gravity collected on R/V Maurice Ewing cruise 9008, October 1990, shows an offset of 2.25 mgals, compared to an average offset of 11.24 mgals for all the cruises. In order to include ship gravity in the derivation of a gravity field, we used the following procedure to adjust the ship data. For each cruise with more than 100 gravity measurements, a quadratic polynomial in time was determined that best fit the differences between the satellite-only gravity anomalies and the ship gravity on a least-squares basis, and this was then subtracted from the ship gravity. The agreement between ship and satellite gravity is then good except for short-wavelength differences that primarily occur at gaps in the altimeter coverage. The adjusted ship gravity was then included with the altimeter gradients to derive a second gravity field. Fig. 3 also shows a comparison between the combined solution and the adjusted gravity along cruise c2115. It is clear that the adjusted shipborne gravity anomalies are not independent of the combined field since they are used in the solution. The following test was made to help evaluate the basic noise level. We constructed a combined gravity field for the area 60°N to 65°N and 328° to 338° in longitude using every other ship gravity value, a total of 7145 points. The comparison between this combined field with the adjusted ship gravity not used in the combined solution (7148 values) yields a mean and rms difference of -0.07 and 2.31 mgals respectively. Before adjustment, the mean and rms differences between ship and satellite-only gravity



**Figure 3.** Ship gravity before and after adjustment compared with satellite-only and combined gravity fields along cruise c2115. The adjusted ship gravity and combined solution are almost coincident.



**Figure 4.** Color image of gravity anomalies from combined satellite and ship data. The image is shaded relief, with illumination and viewpoint of the observer from the southeast.

anomalies are  $-3.84$  and  $11.69$  mgals, while after adjustment the differences are  $0.00$  and  $8.20$  mgals. The mean and rms differences between the adjusted ship and the gravity field derived from both altimeter and ship data are  $-0.05$  and  $2.25$  mgals. The latter number is similar to that in the quality test described above. A colour image of the combined gravity field over the Reykjanes Ridge is shown in Fig. 4.

## Discussion

The transition from axial high to median valley is well-defined in the gravity field in Fig.4, occurring at about  $59^{\circ}\text{N}$  and  $328^{\circ}$ . North of this latitude, a series of highs and lows can be clearly seen fanning out on either side of the Reykjanes Ridge - the V-shaped ridges described by Vogt (1971) - between  $59^{\circ}\text{N}$  to  $64^{\circ}\text{N}$ . Further south, off-ridge gravity anomalies are more coherent and of greater amplitude than to the north. A complete discussion and interpretation of the features revealed in the combined gravity field for the Reykjanes Ridge will be presented elsewhere, but the wealth of coherent short-wavelength detail apparent in Fig. 4 underscores the value of using all available gravity information. The ERS-1 168-day repeat mission, commencing in 1994, will have a 16 km cross-track spacing at the equator, somewhat poorer resolution than the 12-18 km for Geosat/GM (McAdoo and Marks, 1992). However, by combining these 176-day data with existing altimeter data and ship gravity data, we will be able to achieve an overall resolution which is comparable to, if not better than, that of Geosat/GM.

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Chenway Hwang, Department of Civil Engineering, National Chiao Tung University, Hsinchu, Taiwan  
 Barry Parsons, Theresa Strange and Andrew Bingham, Department of Earth Sciences, Oxford University, Oxford, UK

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