

Time-Variant Adjustment for a Level Network

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Abstract: It has been routine work to determine the elevations of points on Earth's surface for a wide range of purposes. However, surveyors are now facing new challenges because of the increasing demand for high-quality measurements despite potential errors caused by Earth's dynamics, which introduce time-dependent signals in the points being investigated. Classical level network adjustment works well when benchmark elevations are known precisely and remain constant over time. However, the traditional approach fails to consider possible ground motions in a single process of network adjustment. This study proposes a time-variant model and unified least-squares technique for level network adjustment. The main emphasis of this approach is to accommodate the time-dependent behaviors of benchmarks caused by ground motions and other unknown sources. The numerical results from a simulated test and a real case study show that the proposed approach can capture the time-dependent signals of benchmarks. Consequently, significant perturbations in benchmarks can be identified, and an improved analysis for level network adjustment thus can be achieved. DOI: [10.1061/\(ASCE\)SU.1943-5428.0000128](https://doi.org/10.1061/(ASCE)SU.1943-5428.0000128). © 2014 American Society of Civil Engineers.

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Introduction

Determining ground-point elevations with respect to a specified reference surface (e.g., geoid or reference ellipsoid) is routinely carried out because it provides the geometric relationships between points in the vertical direction to the local surface. Researchers have used several techniques, including spirit leveling, trigonometric leveling, barometric leveling, and gravimetric leveling, to obtain the necessary observables when using traditional geodetic devices to determine elevations for various purposes (Anderson and Mikhail 2000; Kavanagh 2004; Ghilani and Wolf 2011). In recent years, the Global Navigation Satellite System (GNSS) technique also has been used in leveling surveys because it is considered to be an efficient surveying method for wide-area applications (El-Mowafy et al. 2006; Ustun and Demirel 2006). However, it should be noted that the GNSS technique precisely determines only ellipsoidal heights (vertical distances to a reference ellipsoid). The orthometric heights (vertical distances to a local geoid), which are more widely useful, especially in engineering applications, also can be obtained by the GNSS technique only if an accurate geoid model with a proper corrector surface model is available (see, e.g., Fotopoulos et al. 2003; Wang et al. 2012; Hwang et al. 2013). Despite the existence of various techniques applicable to a leveling survey, the techniques are primarily performed in a relative

sense, meaning that the difference in elevation is measured first. Then, by connecting the measurements to one or more reference points whose elevations are precisely known (also referred to as benchmarks), the elevations of newly measured points are determined accordingly. Furthermore, to ensure the quality of height determination, a leveling survey usually establishes a network containing multiple intersecting lines of levels tied to one or more benchmarks to achieve higher accuracy (Fig. 1). Many constraints thus can be formed (e.g., the summation of the observed elevation differences in any closed loop should be zero, or the total observed elevation difference between any two benchmarks should be equal to the difference in their known elevations) so that possible measurement errors can be minimized and adjusted for. The validation of these constraints necessitates known constant elevations of the benchmarks. The quality of a leveling survey is then evaluated based on how well these constraints are fulfilled (e.g., the elevational misclosure per square root of level-line distance).

On the other hand, as a result of global tectonic motions and local geodynamics, it is now a widely accepted fact that any point on the Earth's surface is moving with respect to any other point (DeMets et al. 1990). This can produce consistent movement when the tectonic plate is relatively rigid (e.g., the North American plate interior) (see Calais et al. 2006). However, inconsistent movements also can be introduced for the ground points if the plate is not rigid and is homogeneously or non-homogeneously deformed (e.g., the regions in a plate boundary) (see Yu et al. 1997; Han et al. 2011). In this case, the relative positions between points do not remain constant, and a time-dependent formulation is required to model these spatial variations. In recent years, researchers have performed extensive studies to determine the time-dependent behaviors of the ground points and terrestrial reference frames affected by Earth's dynamics (Soler 1998; Altamimi et al. 2002; Soler and Marshall 2003; Han and van Gelder 2006). Rigorous models and software also have been developed so that one can easily predict the position of a point at any specific epoch (Pearson et al. 2010). Nevertheless, it should be noted that the preceding modeling of ground-point movements is based primarily on the GNSS technique because it continuously provides spatial measurements for the points of interest so that their time-dependent signals can be captured precisely. As a consequence, the resulting time-dependent model is typically valid for the frames tied to the GNSS (e.g., the global ellipsoidal or global

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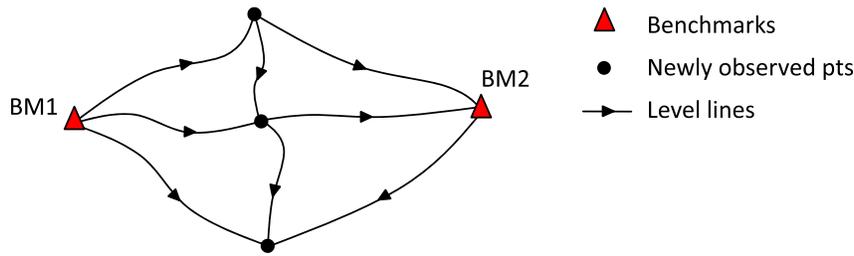


Fig. 1. Level network with multiple level lines and benchmarks

Cartesian frame) but has not been widely applicable for other unrelated alternatives (e.g., the frame of the local orthometric height) unless their relations to the GNSS frames can be determined precisely. Clearly, the benchmarks in a level network are also subject to ground motions, causing their positions to vary over time. Failing to properly consider these time-dependent variations introduces bias into the adjustment of a level network. This becomes particularly problematic if the network contains multiple benchmarks with spatially variable vertical velocities.

This study proposes a time-variant model for adjusting a level network. This method is based on the assumption that different benchmarks move at different velocities and that differential leveling measurements are made at arbitrary epochs. By employing a unified approach to least-squares adjustment, time-dependent behaviors of benchmarks and the elevations of newly observed points can be determined simultaneously. This study also presents a simulation test and a case study based on an actual leveling data set so that the performance of the proposed approach can be evaluated.

Mathematical Model

The mathematical model for height differences in a level-network adjustment is (Anderson and Mikhail 2000)

$$\Delta h_{AB} = h_B - h_A \quad (1)$$

where Δh_{AB} = denotes the observed elevation difference between any two points A and B in the network with elevations that are h_A and h_B , respectively. These elevations are given as known constants if they refer to benchmarks. For other points, their elevations h are treated as unknown parameters to be estimated from the adjustment computation. Two requirements should be fulfilled to uniquely solve for a level network with u points whose elevations are unknown: (1) there should be at least one benchmark elevation to provide an absolute datum for the network, and (2) there should be at least u elevation-difference observations connecting all the points in the network. An ordinary-least-squares (OLS) adjustment technique then can be applied to estimate the unknown elevations. This approach typically works well on the basis that the given heights (i.e., known elevations of benchmarks) are reliable. However, this model is valid only when the observations and elevations of benchmarks are defined in the same epoch. When observations are made in different epochs, the change in benchmark elevations will cause errors in the adjustment model. To accommodate the possible variations in point elevations over time, this observation equation should be revised and extended in a time-variant form. First, given a differential level observation Δh between any two benchmarks i and j , one can use the following mathematical model:

$$\begin{aligned} \Delta h_{ij,t_k} &= [\bar{h}_{j,t_j} + v_j(t_k - t_j)] - [\bar{h}_{i,t_i} + v_i(t_k - t_i)] \\ &= (\bar{h}_{j,t_j} - \bar{h}_{i,t_i}) + v_j(t_k - t_j) - v_i(t_k - t_i) \end{aligned} \quad (2)$$

where \bar{h}_{i,t_i} and \bar{h}_{j,t_j} = elevations of benchmarks i and j , which are defined at the epochs t_i and t_j , respectively; v_i and v_j = their first-order time derivatives (i.e., vertical velocities); and t_k = the epoch when the elevation difference $\Delta h_{ij,t_k}$ is observed. It should be noted that all the input observables (elevation differences) are made in a relative sense, so they do not provide any absolute datum constraint. Datum information (\bar{h}_{i,t_i} , \bar{h}_{j,t_j} , and at least one known vertical velocity corresponding to a specific frame) must be given to determine the time-dependent variation of each benchmark's elevation. This required information can be errorless or contain some level of uncertainty. Consequently, there will be $n - 1$ unknown parameters (v_i , $i = 1 - n$ with one given) to be solved for a level network with n previously positioned benchmarks. On the other hand, the maximum number of elevation-difference observations between these n benchmarks is C_2^n . This produces a maximum number of degrees of freedom $r = C_2^n - n + 1$. As long as $r \geq 0$, a least-squares adjustment can be applied to find the velocity parameters under the condition that the given benchmark elevations (\bar{h}_{i,t_i} and \bar{h}_{j,t_j}) are known precisely or subject to some level of uncertainty ($\pm \sigma_{\bar{h}_i}$ and $\pm \sigma_{\bar{h}_j}$).

This equation also can be used to relate the elevation difference between a previously positioned benchmark whose elevation at t_i equals \bar{h}_{i,t_i} and a newly observed point I , which is then written as

$$\Delta h_{I,t_k} = h_{I,t_k} - [\bar{h}_{i,t_i} + v_i(t_k - t_i)] \quad (3)$$

or between two newly observed points I and J as

$$\Delta h_{IJ,t_k} = h_{J,t_k} - h_{I,t_k} \quad (4)$$

For each additional newly measured point, it is possible to make one or more elevation observations associated with other previously positioned or newly observed points. This makes it relatively easy to achieve a positive redundant number for Eqs. (3) and (4) in practical applications. Finally, these three types of equations can be integrated into a least-squares adjustment model (where a positive number of degrees of freedom can be achieved for each type of equation) so that both the time-dependent elevation parameters at previously positioned benchmarks and the elevations of newly measured points can be solved simultaneously.

In these equations, the elevations of benchmarks are not necessarily constants but can be known values with a certain level of uncertainty. This is practically true because the benchmark elevations are usually obtained from a previous network adjustment (presumably with a higher order of accuracy). They should inherit the uncertainties from the observables in the previous adjustment computations. As a result, the benchmark elevations are not fixed constants, and the adjustment

process should allow corrections that lower their uncertainties. To fulfill this requirement, this study adopts the following unified approach to least-squares adjustment (see Mikhail and Ackermann 1976): First, the observation equations are written in matrix representations as

$$\begin{aligned} \mathbf{l} + \mathbf{v} &= \mathbf{A}\mathbf{\Delta}_x + \mathbf{f} \\ \mathbf{l}_x + \mathbf{v}_x &= \mathbf{x}_0 + \mathbf{\Delta}_x \end{aligned} \quad (5)$$

where \mathbf{l} = observation vector that contains all three types of differential elevation measurements in Eqs. (2)–(4); \mathbf{v} = its residual vector; and $\mathbf{\Delta}_x$ = correctional vector for all parameters (\bar{h}_{i,t_i} , \bar{h}_{j,t_j} , v_i , v_j , h_{j,t_k} , and h_{i,t_k}) presented in the model. Note that correction terms to the previously positioned benchmarks are included in $\mathbf{\Delta}_x$. \mathbf{A} and \mathbf{f} , respectively, represent the design matrix and constant vector, whereas \mathbf{l}_x and \mathbf{v}_x are the pseudo-observation and residual vectors, respectively, for the parameter vector \mathbf{x} , with an initial value denoted by \mathbf{x}_0 . The least-squares estimate for the parameter correction vector can be obtained by

$$\mathbf{\Delta}_x = (\mathbf{N} + \mathbf{W}_{xx})^{-1}(\mathbf{t} - \mathbf{W}_{xx}\mathbf{f}_x) \quad (6)$$

where $\mathbf{N} = \mathbf{A}^T\mathbf{P}\mathbf{A}$; $\mathbf{t} = \mathbf{A}^T\mathbf{P}\mathbf{f}$; $\mathbf{f}_x = \mathbf{x}_0 - \mathbf{l}_x$; \mathbf{P} = weight matrix for the observation vector \mathbf{l} ; and \mathbf{W}_{xx} = a priori weight matrix for the parameter vector \mathbf{x} . Generally, it is considered that the weights of the differential level lines are inversely proportional to their lengths (see Ghilani and Wolf 2006). Consequently, for a level line l whose length is S_l , the associated weight can be determined by

$$p_l \propto \frac{1}{S_l} \quad (7)$$

Furthermore, because three types of parameters (i.e., elevations and vertical velocities at previously positioned benchmarks and elevations at other observed points) could be present in the model, the parameter weight matrix can be written in block diagonal form as

$$\mathbf{W}_{xx} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W}_3 \end{bmatrix} \quad (8)$$

where \mathbf{W}_1 , \mathbf{W}_2 , and \mathbf{W}_3 = weight matrices for the elevations and vertical velocities at previously positioned benchmarks and the elevations at other observed points, respectively. The vertical velocities at benchmarks and elevations at observed points are unknown values before adjustment and are therefore assigned an infinitely small weight matrix (i.e., $\mathbf{W}_2 \cong \mathbf{W}_3 \cong \mathbf{0}$). For the elevations at given benchmarks, the a priori weight matrix can be constructed according to the nominal uncertainties of the benchmarks as

$$\mathbf{W}_1 = \sigma_0^2 \sum_{hh}^{-1} = \sigma_0^2 \begin{bmatrix} \sigma_{\bar{h}_1}^2 & \sigma_{\bar{h}_1\bar{h}_2} & \cdots & \sigma_{\bar{h}_1\bar{h}_u} \\ & \sigma_{\bar{h}_2}^2 & \cdots & \sigma_{\bar{h}_2\bar{h}_u} \\ & & \ddots & \vdots \\ \text{symm.} & & & \sigma_{\bar{h}_u}^2 \end{bmatrix}^{-1} \quad (9)$$

where $\sigma_{\bar{h}_i}$ and $\sigma_{\bar{h}_i\bar{h}_j}$ = SD and covariance values of the previously positioned elevation values at benchmarks, respectively; and σ_0^2 = variance of unit weight. After obtaining the correction vector, the parameter estimates can be computed as

$$\hat{\mathbf{x}} = \mathbf{x}_0 + \mathbf{\Delta}_x \quad (10)$$

and its variance-covariance matrix is evaluated by

$$\mathbf{\Sigma}_{\hat{\mathbf{x}}\hat{\mathbf{x}}} = \hat{\sigma}_0^2 (\mathbf{N} + \mathbf{W}_{xx})^{-1} \quad (11)$$

where $\hat{\sigma}_0$ = a posteriori SD of the unit weight, which can be estimated by

$$\hat{\sigma}_0 = \sqrt{\frac{\mathbf{v}^T\mathbf{P}\mathbf{v} + \mathbf{v}_x^T\mathbf{W}_{xx}\mathbf{v}_x}{r}} \quad (12)$$

With these equations, all the parameters (including the elevations and vertical velocities at previously positioned benchmarks and the elevations at newly observed points) and their associated uncertainties can be determined. However, one should be reminded that the proposed model deals only with the case in which the time-variant signals are primarily the result of persistent movements of benchmarks. Other factors, including errors in the previously computed elevations of benchmarks and uncorrected systematic errors in leveling observations (e.g., curvature of the Earth, effect of refraction, geopotential variation, etc.), could alias into the estimated velocities. A careful check of the data set always should be performed before it is analyzed by the proposed approach.

Numerical Validation

Simulation Test

This section presents a simulated level network with four previously computed benchmarks and six newly observed points (Fig. 2). The four benchmarks with previously positioned elevations and uncertainties are defined at different epochs with various vertical velocities (Table 1). The elevation-difference observations at epoch

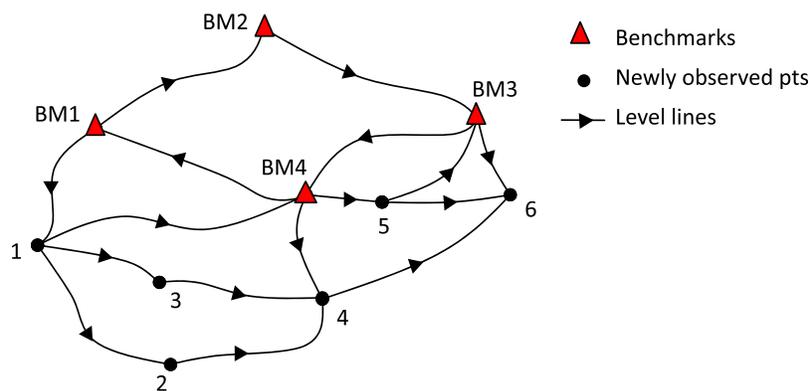


Fig. 2. Simulated level network used for analysis

Table 1. Simulated Benchmarks Used in Analysis

Site ID	Elevation (m)	SD (m)	Epoch (year)	Vertical velocity (m/year)
BM1	100.002	±0.001	2009.0	0.020
BM2	102.003	±0.002	2009.0	-0.010
BM3	103.502	±0.003	1998.0	0.003
BM4	99.865	±0.002	2007.0	0.015

Table 2. Simulated Differential Elevation Measurements at Epoch 2012.0

Backsight	Foresight	Observed elevation difference (m)
BM1	BM2	1.911
BM2	BM3	1.568
BM3	BM4	-3.598
BM4	BM1	0.127
BM1	1	-3.729
1	BM4	3.613
1	2	1.639
1	3	9.306
2	4	-5.397
3	4	-13.072
BM4	4	-7.381
4	6	1.561
BM4	5	0.067
5	6	-5.877
5	BM3	3.542
BM3	6	-9.421

2012.0 with 3-mm random errors also were simulated and are listed in Table 2. Because there are 16 elevation-difference observations with 6 elevations to be determined, the redundancy number of this network is 10.

Based on these observables, this study presents estimates of the elevations of the newly measured points for two cases. Case 1 is a classical level network that does not consider the time-dependent variations of benchmarks (i.e., a static model). Case 2 considers the first-order time-dependent variation of benchmarks (i.e., a time-variant model). In both cases, all the elevation-difference observations are assumed to have an equal level of quality (± 3 mm). The model is used to construct the associated weight matrix for input observations. The known elevations of all four benchmarks (BM1–BM4) and their uncertainties are also used in the adjustment to provide loosely a datum constraint for the network. Additionally, in Case 2, the vertical velocity is assumed to be known and fixed for BM1 but is treated as unknown for the other benchmarks during the adjustment computation. Table 3 lists the adjusted elevations and associated statistics based on the observations at epoch 2012 for both cases. The estimated elevation rates at the benchmarks based on the time-variant elevation model are listed in Table 4.

These results indicate that neglecting the time-variant variations in the benchmarks leads to a degraded result. This is so because the elevation variations of the benchmarks introduce systematic biases into the measurements. On the other hand, a time-variant elevation model can accurately identify the time-dependent variations of the benchmarks. Consequently, the resulting adjustment results became more reliable. Furthermore, one also should notice that the elevations of the benchmarks after the adjustment are not necessarily identical to their a priori values. This is due to the fact that the uncertainties of the previously positioned elevations of the benchmarks have been considered in the model, and the adjustment allows

Table 3. Adjusted Elevations for Simulated Network at Epoch 2012

Site ID	Case 1: Static model		Case 2: Time-variant model	
	Adjusted elevation (m)	SD (m)	Adjusted elevation (m)	SD (m)
BM1	100.007	±0.006	100.062	±0.001
BM2	101.967	±0.010	101.973	±0.003
BM3	103.502	±0.012	103.544	±0.004
BM4	99.882	±0.010	99.940	±0.003
1	96.272	±0.014	96.328	±0.003
2	97.908	±0.019	97.963	±0.004
3	105.579	±0.019	105.634	±0.004
4	92.509	±0.016	92.562	±0.004
5	99.954	±0.016	100.002	±0.004
6	94.076	±0.016	94.123	±0.004
$\hat{\sigma}_0$	6.838		1.266	
RMSV ^a (m)	0.016		0.003	

^aRMS residuals.

Table 4. Estimated Vertical Velocities at Benchmarks Based on Time-Variant Elevation Model

Site ID	Vertical velocities (m/year)	SD (m/year)
BM1	0.020 ^a	N/A
BM2	-0.010	±0.002
BM3	0.003	±0.000
BM4	0.015	±0.001

^aFixed during adjustment.

corrections to these a priori values [Eq. (5)]. This reveals one of the characteristics of the unified least-squares adjustment technique adopted in this study.

Case Study Based on a Real Data Set

A first-order, class II (see Ministry of the Interior 2001) level network in Taipei City was investigated. This network consists of 6 reference benchmarks and 24 points whose orthometric heights are to be determined using a differential leveling. Thirty-six level lines (conforming to an accuracy standard of $0.7 \text{ mm}/\sqrt{\text{km}}$) were surveyed in 2012 to obtain the elevation differences between these 30 points (Fig. 3). The redundancy number of this network was 12. The Department of Urban Development, Taipei City Government, provided the known elevations of the six reference benchmarks, which were determined previously at two epochs (in 2003 and 2009). Table 5 lists their published elevations and uncertainties.

Before a further analysis was undertaken, a free-net adjustment was carried out on the original 2012 data set without using any known elevations of the benchmarks. The measurements that did not fulfill the accuracy standard were identified as blunders and removed from any subsequent analysis so that the internal consistency of the data set could be assured. Additionally, the authors did not perform corrections for the curvature of the Earth or the effects of refraction and geopotential variations in this particular case because the network covered only a limited area ($<3 \times 5$ km), and the averaged length of the level lines was 0.45 km. The authors assume that the associated errors are not significant or have been mostly eliminated through proper field procedures (e.g., balancing the backsight and foresight distances). Table 6 summarizes the elevation misclosures for all the loops in the network. The misclosure values (ranging from

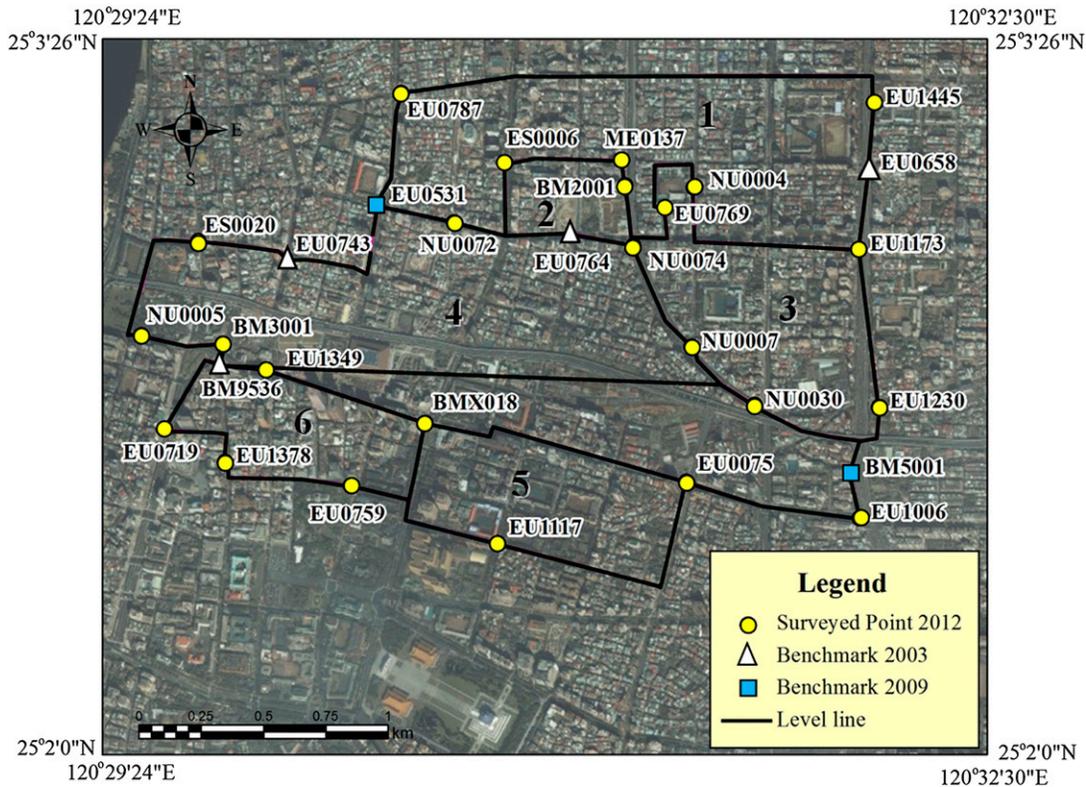


Fig. 3. Level network in Taipei City used for case study (data from Esri, DigitalGlobe, GeoEey, I-Cubed, USDA, USGS, AEX, Getmapping, Aerogrid, IGN, IGP, swisstopo, and the GIS USER Community)

Table 5. Reference Benchmarks Used in Case Study

Site ID	Elevation (m)	SD (mm)	Epoch (year)
EU0743	3.016	±1.02	2003.0
EU0531	2.957	±0.99	2009.0
EU0764	3.838	±1.00	2003.0
EU0658	4.844	±1.06	2003.0
BM5001	5.762	±1.02	2009.0
BM9536	4.967	±1.01	2003.0

Table 6. Loop Misclosures Based on Observations Used in Case Study

Loop ID	Misclosure (m)
1	-0.004
2	-0.001
3	0.002
4	0.003
5	0.001
6	0.002

-4 to 3 mm) indicate good internal consistency of the input observations prior to the subsequent adjustment analyses.

To assess the proposed approach, two cases (one with the static model and the other with the proposed time-variant model) were again analyzed, but now with the actual data set. In Case 2, the vertical velocity of EU0531 is assumed to be zero, and therefore, the vertical velocities of other points are determined relative to this point. Table 7 lists the adjusted elevations for these two cases, and Table 8 shows the determined vertical velocities of the reference benchmarks.

Table 7 shows that the proposed time-variant model improved the adjustment results for the investigated network. The postadjustment SD of unit weight became closer to unity, and the RMS residual was approximately 0.43 mm in Case 2. This is a more reliable result than the classical approach because it realistically reflects the quality of the input elevation-difference observations ($0.7 \text{ mm}/\sqrt{\text{km}}$ with a 0.45-km average length of level lines). These results also show significant vertical velocities (ranging from -4.06 to 1.82 mm/year) of the reference benchmarks relative to point EU0531, which reveals that there must be apparent ground motions at these points relative to EU0531 during the period of investigation. Furthermore, the benchmark EU0764 is subjected to a maximal value of ground subsidence in the network. Consequently, the points near EU0764 are greatly perturbed, and their adjusted elevations are significantly varied after implementing the proposed time-variant adjustment approach. Finally, the adjusted elevation values for the benchmarks differ from their a priori values. The reason was discussed in the preceding case study using the simulated data set. In summary, this case study illustrated that accurate leveling observations do not necessarily lead to a high-quality height determination unless the time-dependent variations of the reference points are properly taken into account during the adjustment treatment.

Remarks

Fostered by rapid developments in spatial sciences and technologies, the demand for a high-quality positioning method on Earth's surface is constantly growing. To achieve high fidelity, the time-dependent signals produced by Earth's dynamics should not be overlooked. This study proposes a time-variant model for adjusting a level network so that a simultaneous process of previously

Table 7. Adjusted Elevations for Investigated Network at Epoch 2012

Site ID	Case 1: Static model		Case 2: Time-variant model	
	Adjusted elevation (m)	SD (mm)	Adjusted elevation (m)	SD (mm)
EU0743	3.009	±5.71	2.999	±1.39
EU0531	2.967	±5.09	2.957	±1.15
EU0764	3.818	±5.25	3.802	±1.16
EU0658	4.846	±5.57	4.837	±1.23
BM5001	5.773	±5.68	5.768	±1.18
BM9536	4.970	±5.88	4.962	±1.17
BM3001	4.744	±6.16	4.736	±1.59
EU1349	4.717	±6.48	4.709	±1.63
EU0787	2.210	±5.99	2.199	±1.32
ES0020	3.205	±6.38	3.195	±1.47
NU0072	4.181	±5.96	4.169	±1.28
ES0006	3.150	±6.23	3.137	±1.38
NU0074	5.977	±5.22	5.964	±1.41
ME0137	4.327	±5.55	4.316	±1.40
BM2001	5.624	±5.57	5.612	±1.40
NU0007	5.731	±6.10	5.720	±1.51
NU0030	5.499	±5.89	5.490	±1.52
EU0075	5.158	±6.14	5.150	±1.56
BMX018	5.560	±6.61	5.552	±1.61
EU1445	4.387	±5.97	4.377	±1.51
EU1173	4.940	±5.74	4.931	±1.52
NU0004	3.888	±6.40	3.877	±1.52
EU0769	4.537	±6.05	4.524	±1.48
EU1230	5.585	±5.69	5.578	±1.54
EU1006	5.480	±6.14	5.474	±1.62
EU1117	4.186	±7.19	4.178	±1.64
NU0005	4.237	±6.53	4.228	±1.56
EU0719	4.879	±7.07	4.871	±1.67
EU1378	4.089	±7.58	4.081	±1.71
EU0759	4.779	±7.55	4.771	±1.69
$\hat{\sigma}_0$	8.51		1.16	
RMSV ^a (mm)	2.38		0.43	

^aRMS residuals.**Table 8.** Estimated Vertical Velocities at Reference Benchmarks in Investigated Network

Site ID	Vertical velocities (mm/year)	SD (mm/year)
EU0743	-1.94	±0.21
EU0531	0.00 ^a	N/A
EU0764	-4.06	±0.20
EU0658	-0.83	±0.21
BM5001	1.82	±0.66
BM9536	-0.59	±0.22

^aFixed during adjustment.

determined benchmark elevations and newly collected leveling observables can be achieved. The numerical results of a simulation test and a real case study show that the proposed approach can properly capture the time-variant signals of benchmarks in a level network and thus minimize the inconsistencies between previously published elevations and new leveling observables. However, it also should be noted that without the supporting evidence from long-term continuous field observations, the time-variant signals obtained at benchmarks cannot be simply attributed to ground motions but can

only be considered as the numerical difference between old published and newly determined benchmark elevation values. Other sources or undetected errors in the old leveling data also may result in perturbation signals. The results from the proposed approach can be used on an immediate basis to identify significantly perturbed benchmarks, and further arrangements thus can be introduced (e.g., collecting other measurements to confirm the ground motions at the candidate benchmarks and/or applying adequate constraints in a subsequent network analysis). On the other hand, if one needs to fully understand the time-dependent behavior of a level network in a scientific manner, a much more complicated procedure is required (e.g., including the reanalysis of all historical leveling data, performing long-term continuous field observations, and constructing appropriate models for all the factors affecting a level network). Clearly, this type of procedure involves tremendous cost and time and is beyond the scope of this study. Interested readers are redirected to related works such as Hwang et al. (2008), and Hung et al. (2010, 2011). Finally, the proposed approach estimates only the persistent vertical variations (i.e., the first-order time derivatives of vertical positions) of benchmarks across a time frame. The fluctuations caused by short-term periodic signals may not be well accommodated and still can cause fractional noise in a level network. An additional model that considers all the local recurring noises, when available, should be integrated into the proposed approach if these short-term signals cause significant perturbations. Undoubtedly, subsequent studies should keep exploring the possibility of further extending the time-variant model so that the quality of a level network will be continuously improved.

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